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全反射对偏振态的改变规律及实验验证

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摘 要:基于菲涅耳公式,推导出一束偏振光经过全反射后偏振状态的改变规律,定量计算 s 光和 p 光的相位变化以及振幅变化,给出理论公式. 并提出一种将光矢量转化为光强的思想,设计实验,以线偏振光为例验证理论公式的正确性,得到全反射对偏振态的改变规律.

关键词:全反射:菲涅耳公式:振幅:相位差

中图分类号: O436.3

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一東光经过全反射后的偏振状态会发生改变,运用菲涅耳公式可以推导出入射角、折射率等对偏振状态影响的理论公式.已有文献从纯数学、矩阵等等角度出发推导出公式,但少有实验方法直观地验证此规律.本文将光矢量转化为光强,以最简单的线偏振光、45°夹角为例,设计实验验证.

1 理论依据

全反射光路如图 1 所示,A 代表入射光,A' 代表反射光, n_i 为入射光所在介质的折射率, n_2 为折射光所在介质的折射率,研究全反射过程,故有 $n_i > n_2$.

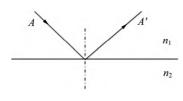


图 1 全反射光路

由菲涅耳公式再结合反射定律可以得到[1]:

$$\mathbf{E_{s_1}}' = \frac{\mathbf{n_{12}}\cos \mathbf{i_1} + \mathbf{i} \ \sqrt{(\mathbf{n_{12}}\sin \mathbf{i_1})^2 - 1}}{\mathbf{n_{12}}\cos \mathbf{i_1} - \mathbf{i} \ \sqrt{(\mathbf{n_{12}}\sin \mathbf{i_1})^2 - 1}}\mathbf{E_{s_1}}, (1)$$

$$\mathbf{E}_{p_1}' = \frac{\cos i_1 + n_{12} i \sqrt{(n_{12} \sin i_1)^2 - 1}}{\cos i_1 - n_{12} i \sqrt{(n_{12} \sin i_1)^2 - 1}} \mathbf{E}_{p_1}, (2)$$

其中, E 为光的复振幅形式, $n_{12} = n_1/n_2$.

1.1 全反射后的振幅变化 对(1)式取模得:

$$|\mathbf{E}_{s_1}'| = \left| \frac{\mathbf{n}_{12} \cos \mathbf{i}_1 + \mathbf{i} \ \sqrt{(\mathbf{n}_{12} \sin \mathbf{i}_1)^2 - 1}}{\mathbf{n}_{12} \cos \mathbf{i}_1 - \mathbf{i} \ \sqrt{(\mathbf{n}_{12} \sin \mathbf{i}_1)^2 - 1}} \right| |\mathbf{E}_{s_1}| = \frac{|\mathbf{n}_{12} \cos \mathbf{i}_1 + \mathbf{i} \ \sqrt{(\mathbf{n}_{12} \sin \mathbf{i}_1)^2 - 1}|}{|\mathbf{n}_{12} \cos \mathbf{i}_1 - \mathbf{i} \ \sqrt{(\mathbf{n}_{12} \sin \mathbf{i}_1)^2 - 1}|} |\mathbf{E}_{s_1}| = |\mathbf{E}_{s_1}|.$$

同理对(2)式取模得:

$$|E_{p_1}'| = |E_{p_1}|.$$

因此,全反射不改变振幅.

1.2 全反射后的相位变化(以线偏振光为例)

将振幅写成复振幅形式(记入射光初相位为0,即入射光 s 光和 p 光相位差为 0),经推导可得反射光的相位满足以[1]

$$\tan \varphi_{s} = \frac{2 n_{12} \cos i_{1} \sqrt{(n_{12} \sin i_{1})^{2} - 1}}{(n_{12} \cos i_{1})^{2} - (n_{12} \sin i_{1})^{2} + 1}, \quad (3)$$

$$\tan \varphi_{p} = \frac{2 n_{12} \cos i_{1} \sqrt{(n_{12} \sin i_{1})^{2} - 1}}{(\cos i_{1})^{2} - n_{12}^{2} [(n_{12} \sin i_{1})^{2} - 1]}. \quad (4)$$

因此可得到:

- 1)相位变化只与相对折射率 n_{12} 和入射角 i_1 有关;
 - 2)反射光 s'光和 p'光均较入射光变"快"(即

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s'光和 p'光较于 s 光和 p 光相位差为正值),其中 s'光和 p'光较入射光的相位差变化范围是: $0 \sim \pi$ (对应的入射角是全反射的临界角到 $\pi/2$).

3)反射光 s'光和 p'光的相位差为

$$\varphi_{b}' - \varphi_{s}' = \arctan \left[\frac{2 n_{12} \cos i_{1} \sqrt{(n_{12} \sin i_{1})^{2} - 1}}{(\cos i_{1})^{2} - n_{12}^{2} [(n_{12} \sin i_{1})^{2} - 1]} \right] - \arctan \left[\frac{2 n_{12} \cos i_{1} \sqrt{(n_{12} \sin i_{1})^{2} - 1}}{(n_{12} \cos i_{1})^{2} - (n_{12} \sin i_{1})^{2} + 1} \right],$$

即线偏振光经过全反射之后,变成椭圆偏振光.

 φ_{s} '和 φ_{s} '随入射角 i 的变化如图 2 所示,s '光和 p '光之间的相位差(φ_{s} '一 φ_{s} ')随入射角 i 的变化如图 3 所示.

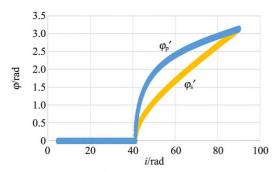


图 2 φ, '和 φ, '随入射角 i 的变化

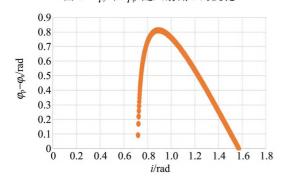


图 3 相位差 $\mathbf{q}_i - \mathbf{q}_i$ 随入射角 i 的变化

1.3 实验测量相位变化(以线偏振光为例)

实验无法直接测量光矢量,因此,需将光矢量 转化为光强.本文采取起偏器起偏得线偏振光, 检偏器检测光强的方法.

经过起偏器起偏,如图 4 所示得到:

$$A_s = A \sin a, A_o = A \cos a,$$

为方便实验探究,取 $a=45^{\circ}$,即:

$$\mathbf{A}_{s} = \mathbf{A}\sin \mathbf{a} = \frac{\sqrt{2}}{2}\mathbf{A},$$

$$\mathbf{A}_{p} = \mathbf{A}\cos \mathbf{a} = \frac{\sqrt{2}}{2}\mathbf{A},$$

反射光是2束相干光的叠加,为便于测量,取

检偏器与起偏器平行和垂直方向,测量其光强. 如图 5 所示,其中虚线代表检偏器的方向.

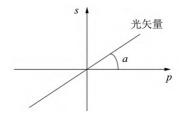


图 4 起检器起偏

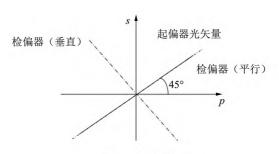


图 5 检偏器检编

根据光强叠加

$$\mathbf{I} = \mathbf{A}_1^2 + \mathbf{A}_2^2 + 2\mathbf{A}_1 \mathbf{A}_2 \cos \Delta \boldsymbol{\varphi},$$

当检偏器与起偏器平行时

$$I_{1} = \mathbf{A}_{s(\hat{\mathbf{k}}_{\mathbf{M}})}^{2} + \mathbf{A}_{p(\hat{\mathbf{k}}_{\mathbf{M}})}^{2} + 2\mathbf{A}_{s(\hat{\mathbf{k}}_{\mathbf{M}})} \mathbf{A}_{p(\hat{\mathbf{k}}_{\mathbf{M}})} \cos \Delta \boldsymbol{\varphi} = (\mathbf{A}_{s}' \sin 45^{\circ})^{2} + (\mathbf{A}_{p}' \cos 45^{\circ})^{2} + 2\mathbf{A}_{s}' \sin 45^{\circ} \mathbf{A}_{p}' \cos 45^{\circ} \cos \Delta \boldsymbol{\varphi} = \frac{1}{2} \mathbf{A}^{2} + \frac{1}{2} \mathbf{A}^{2} \cos \Delta \boldsymbol{\varphi}.$$
 (5)

当检偏器与起偏器垂直时

$$I_{2} = A_{s(\hat{\kappa}_{\hat{\mathbf{M}}})}^{2} + A_{p(\hat{\kappa}_{\hat{\mathbf{M}}})}^{2} - 2A_{s(\hat{\kappa}_{\hat{\mathbf{M}}})} A_{p(\hat{\kappa}_{\hat{\mathbf{M}}})} \cos \Delta \varphi = \frac{1}{2} A^{2} - \frac{1}{2} A^{2} \cos \Delta \varphi, \tag{6}$$

其中:

$$\Delta \boldsymbol{\varphi} = \boldsymbol{\varphi}_{b} - \boldsymbol{\varphi}_{s} =$$

$$\operatorname{arctan} \left\{ \frac{2 \boldsymbol{n}_{12} \cos \boldsymbol{i}_{1} \sqrt{(\boldsymbol{n}_{12} \sin \boldsymbol{i}_{1})^{2} - 1}}{(\cos \boldsymbol{i}_{1})^{2} - \boldsymbol{n}_{12}^{2} [(\boldsymbol{n}_{12} \sin \boldsymbol{i}_{1})^{2} - 1]} \right\} -$$

$$\operatorname{arctan} \left[\frac{2 \boldsymbol{n}_{12} \cos \boldsymbol{i}_{1} \sqrt{(\boldsymbol{n}_{12} \sin \boldsymbol{i}_{1})^{2} - 1}}{(\boldsymbol{n}_{12} \cos \boldsymbol{i}_{1})^{2} - (\boldsymbol{n}_{12} \sin \boldsymbol{i}_{1})^{2} + 1} \right].$$

由(5)和(6)式得:

$$\Delta \varphi = \arccos \left(\frac{\mathbf{I}_1 - \mathbf{I}_2}{\mathbf{I}_1 + \mathbf{I}_2} \right),$$

由此通过实验测得 $\Delta \varphi$.

2 探究全反射入射角和相位差的关系

采用半导体激光器产生部分偏振光,经起偏器起偏后得到与水平方向成 45°的线偏振光,之

后入射到 45°的等腰直角棱镜中实现全反射,最后通过检偏器测量光强.实验装置如图 6 所示.

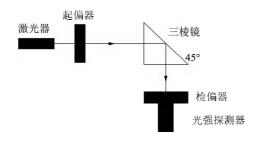


图 6 实验装置图

如图 7 所示,全反射发生在棱镜的斜边上,此时的入射光已经经过了 1 次折射(从空气到棱镜),而根据菲涅耳公式:

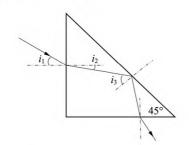


图 7 棱镜内部的传播光路

$$\mathbf{t} = \frac{\mathbf{A}_{s_2}}{\mathbf{A}_{s_1}} = \frac{2 \mathbf{n}_1 \cos \mathbf{i}_1}{\mathbf{n}_1 \cos \mathbf{i}_1 + \mathbf{n}_2 \cos \mathbf{i}_2} = \frac{2 \sin \mathbf{i}_2 \cos \mathbf{i}_1}{\sin (\mathbf{i}_1 + \mathbf{i}_2)},$$

$$\mathbf{t}_{s} = \frac{\mathbf{A}_{s_2}}{\mathbf{A}_{p_1}} = \frac{2 \mathbf{n}_1 \cos \mathbf{i}_1}{\mathbf{n}_1 \cos \mathbf{i}_2 + \mathbf{n}_2 \cos \mathbf{i}_1} = \frac{2 \sin \mathbf{i}_2 \cos \mathbf{i}_1}{\sin (\mathbf{i}_1 + \mathbf{i}_2) \cos (\mathbf{i}_1 - \mathbf{i}_2)},$$
式中, \mathbf{n}_1 为空气的折射率, \mathbf{n}_2 为棱镜的折射率.
因此,折射光不改变相位,只改变振幅,可得

$$\mathbf{A}_{s(fff)} = \mathbf{A}\sin 45^{\circ} \cdot \mathbf{t}_{s},$$
$$\mathbf{A}_{p(fff)} = \mathbf{A}\cos 45^{\circ} \cdot \mathbf{t}_{p},$$

加上折射调制因子后(光从起偏器出射到检偏器 经过2次折射),检偏器与起偏器平行时,

$$\begin{split} \mathbf{I}_{\text{l}} = & \mathbf{A'}_{\text{s}(\hat{\mathbf{K}}\hat{\mathbf{B}})}^{2} + \mathbf{A'}_{\text{p}(\hat{\mathbf{K}}\hat{\mathbf{B}})}^{2} + 2\mathbf{A'}_{\text{s}(\hat{\mathbf{K}}\hat{\mathbf{B}})} \mathbf{A'}_{\text{p}(\hat{\mathbf{K}}\hat{\mathbf{B}})} \cos \Delta \boldsymbol{\varphi} = \\ & \frac{1}{4} \mathbf{A}^{2} (\boldsymbol{t}_{\text{s}}^{\text{l}} + \boldsymbol{t}_{\text{p}}^{\text{l}}) + \frac{1}{2} \mathbf{A}^{2} \boldsymbol{t}_{\text{s}}^{2} \boldsymbol{t}_{\text{p}}^{2} \cos \Delta \boldsymbol{\varphi}; \end{split}$$

检偏器与起偏器垂直时,

$$\begin{split} \mathbf{I}_{2} &= \mathbf{A'}_{\mathrm{s}(\text{檢}_{\mathrm{H}})}^{2} + \mathbf{A'}_{\mathrm{p}(\text{檢}_{\mathrm{H}})}^{2} + 2\mathbf{A'}_{\mathrm{s}(\text{檢}_{\mathrm{H}})} \mathbf{A'}_{\mathrm{p}(\text{檢}_{\mathrm{H}})} \cos \Delta \boldsymbol{\varphi} = \\ &\frac{1}{2} \mathbf{A}^{2} (\mathbf{t}_{\mathrm{s}}^{\mathrm{l}} + \mathbf{t}_{\mathrm{p}}^{\mathrm{l}}) - \frac{1}{2} \mathbf{A}^{2} \mathbf{t}_{\mathrm{s}}^{2} \mathbf{t}_{\mathrm{p}}^{2} \cos \Delta \boldsymbol{\varphi}. \\ &\text{由此得:} \end{split}$$

$$\Delta \boldsymbol{\varphi} = \arccos \left(\frac{\boldsymbol{t}_s^{l} + \boldsymbol{t}_p^{l}}{2 \, \boldsymbol{t}_s^{l} \, \boldsymbol{t}_p^{l}} \, \frac{\boldsymbol{I}_1 - \boldsymbol{I}_2}{\boldsymbol{I}_1 + \boldsymbol{I}_2} \right).$$

对于全反射的入射角,可通过测量经检偏器

起偏的光线与棱镜直角边的夹角,再通过折射定律转化为全反射的入射角,具体关系为

$$i_3 = \arcsin \left(\frac{n_1}{n_2} \sin i_1\right) + \frac{\pi}{4}$$

取空气折射率为 1, 棱镜折射率为 1.52(棱镜 折射率通过测最小偏向角的方法测得),得实验数 据如图 8 所示,与理论值对比如图 9 所示.

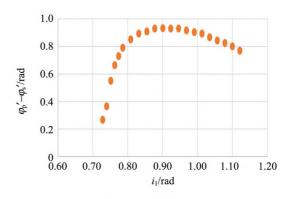


图 8 实验数据图

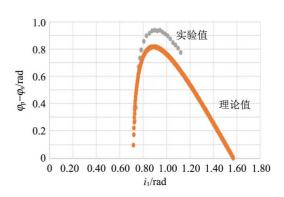


图 9 实验与理论对比

实验数据的趋势和理论值基本一致,但是实验测量值却比理论值偏大(2组实验均发现趋势一致而光程差的数值偏大).对此还未能很好地解释,但是当统一调整了光强的测量值后(光强值统一减去某常数),2条曲线较好地重合了,因此推测原因可能是光强探测器的初值未能很好地调零,从而导致光强的测量值偏大.

3 结 论

1)全反射并不会改变光的振幅,但是会对垂直于人射面的光和平行于人射面的光产生相位变化(相位较于人射光变"大").由于对两者产生的相位变化程度不同,而导致反射光的偏振态与人射光不同.

- 2)当人射光是线偏振光时,其反射光将变成椭圆偏振光,且通常情况下(相对折射率 n_{12} 不大时)由于对垂直于人射面的光和平行于入射面的光之间产生的相位差小于 $\pi/2$,因此不会产生圆偏振光.
- 3) 当入射光是椭圆偏振光时可以进行类似的分析.

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Change rule and its experimental verification of the polarization state in total reflection

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Abstract: The change rule of the polarization state of polarized light after total reflection was studied. The phase difference and amplitude change of s-polarized light and p-polarized light were calculated based on the Fresnel formula. An experiment was designed with linearly polarized light to verify the correctness of the theory. And a method to measure phase difference by converting light vector into light intensity was given.

Key words: total reflection; Fresnel formula; amplitude; phase difference

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Modified model for experiment of measuring sound velocity in water

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Abstract: In the experiment of measuring the sound velocity by the resonance interference method and phase comparison method, the usual model was to consider the transmitted and the reflected ultrasonic wave on the receiving transducer forming standing wave under certain conditions. However, when measuring the sound velocity in water, the experimental phenomenon did not completely agree with the model. On accounting the multiple reflections between the transmitting and receiving transducers, and the effects of the reflection at the water-air interface, a modified model was proposed, and a Mathematica simulation was performed. The results indicated that the depth of water had a great effect on the experimental phenomenon.

Key words: sound velocity measurement; standing wave; resonance interference method; phase comparison method; ultrasonic wave reflection [责任编辑:郭 伟]